
OMLT Documentation

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The OMLT Developers

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OMLT is a Python package for representing machine learning models (neural networks and gradient-boosted trees) within the Pyomo optimization environment. The package provides various optimization formulations for machine learning models (such as full-space, reduced-space, and MILP) as well as an interface to import sequential Keras and general ONNX models.

**CHAPTER
ONE**

INSTALLATION

OMLT requires Python >= 3.6. The most stable OMLT version can be installed using the PyPI package index. This will also install the required dependencies. Simply run:

```
pip install omlt
```

If using the latest un-released version, install from the github repository and install locally

```
git clone https://github.com/cog-imperial/OMLT.git
cd OMLT
pip install .
```

1.1 Optional Requirements

OMLT can import sequential Keras models which requires a working installation of tensorflow:

```
pip install tensorflow
```

OMLT can also import neural network and gradient boosted tree models using ONNX. This requires installing the ONNX interface:

```
pip install onnx
```

CHAPTER TWO

QUICK-START

The quick-start uses the same model shown on the OMLT [README](#), but it provides a little more detail. The example imports a neural network trained in TensorFlow, formulates a Pyomo model, and seeks the neural network input that produces the desired output.

We begin by importing the necessary packages. These include *tensorflow* to import the neural network and *pyomo* to build the optimization problem. We also import the necessary objects from *omlt* to formulate the neural network in Pyomo.

```
import tensorflow
import pyomo.environ as pyo
from omlt import OmltBlock, OffsetScaling
from omlt.neuralnet import FullSpaceNNFormulation, NetworkDefinition
from omlt.io import load_keras_sequential
```

We first load a simple neural network from the tests directory that contains 1 input, 1 output, and 3 hidden nodes with sigmoid activation functions.

```
#load a Keras model
nn = tensorflow.keras.models.load_model('tests/models/keras_linear_131_sigmoid',  
                                         compile=False)
```

We next create a Pyomo model and attach an *OmltBlock* which will be used to formulate the neural network. An *OmltBlock* is a custom Pyomo block that we use to build machine learning model formulations. We also create Pyomo model variables to represent the input and output of the neural network.

```
#create a Pyomo model with an OMLT block
model = pyo.ConcreteModel()
model.nn = OmltBlock()

#the neural net contains one input and one output
model.input = pyo.Var()
model.output = pyo.Var()
```

OMLT supports the use of scaling and input bound information. This information informs how the Pyomo model applies scaling and unscaling to the neural network inputs and outputs. It also informs variable bounds on the inputs.

```
#apply simple offset scaling for the input and output
scale_x = (1, 0.5)      #(mean,stdev) of the input
scale_y = (-0.25, 0.125) #(mean,stdev) of the output
scaler = OffsetScaling(offset_inputs=[scale_x[0]],
                      factor_inputs=[scale_x[1]],
                      offset_outputs=[scale_y[0]],
```

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```

    factor_outputs=[scale_y[1]])

#provide bounds on the input variable (e.g. from training)
scaled_input_bounds = {0:(0,5)}

```

We now create a *NetworkDefinition* using the *load_keras_sequential* function where we provide the scaler object and input bounds. Once we have a *NetworkDefinition*, we can pass it to various formulation objects which decide how to build the neural network within the *OmltBlock*. Here, we use the *FullSpaceNNFormulation*, but others are also possible (see *formulations*).

```

#load the keras model into a network definition
net = load_keras_sequential(nn,scaler,scaled_input_bounds)

#multiple formulations of a neural network are possible
#this uses the default NeuralNetworkFormulation object
formulation = FullSpaceNNFormulation(net)

#build the formulation on the OMLT block
model.nn.build_formulation(formulation)

```

We can query the input and output pyomo variables that the *build_formulation* method produces (as well as scaled input and output varialbes). We lastly create pyomo constraints that connect our input and output variables defined earlier to the neural network input and output variables on the *OmltBlock*.

```

#query inputs and outputs, as well as scaled inputs and outputs
model.nn.inputs
model.nn.outputs
model.nn.scaled_inputs
model.nn.scaled_outputs

#connect pyomo model input and output to the neural network
@model.Constraint()
def connect_input(mdl):
    return mdl.input == mdl.nn.inputs[0]

@model.Constraint()
def connect_output(mdl):
    return mdl.output == mdl.nn.outputs[0]

```

Lastly, we formulate an objective function and use Ipopt to solve the optimization problem.:

```

#solve an inverse problem to find that input that most closely matches the output value
of 0.5
model.obj = pyo.Objective(expr=(model.output - 0.5)**2)
status = pyo.SolverFactory('ipopt').solve(model, tee=False)
print(pyo.value(model.input))
print(pyo.value(model.output))

```

CHAPTER
THREE

JUPYTER NOTEBOOKS

OMLT provides Jupyter notebooks to demonstrate its core capabilities. All notebooks can be found on the OMLT [github page](#). The notebooks are summarized as follows:

- [build_network.ipynb](#) shows how to manually create a *NetworkDefinition* object. This notebook is helpful for understanding the details of the internal layer structure that OMLT uses to represent neural networks.
- [import_network.ipynb](#) shows how to import neural networks from Keras and PyTorch using ONNX interoperability. The notebook also shows how to import variable bounds from data.
- [neural_network_formulations.ipynb](#) showcases the different neural network formulations available in OMLT.
- [mnist_example_dense.ipynb](#) trains a fully dense neural network on MNIST and uses OMLT to find adversarial examples.
- [mnist_example_convolutional.ipynb](#) trains a convolutional neural network on MNIST and uses OMLT to find adversarial examples.
- [auto-thermal-reformer.ipynb](#) develops a neural network surrogate (using sigmoid activations) with data from a process model built using [IDAES-PSE](#).
- [auto-thermal-reformer-relu.ipynb](#) develops a neural network surrogate (using ReLU activations) with data from a process model built using [IDAES-PSE](#).
- [bo_with_trees.ipynb](#) incorporates gradient-boosted-trees into a Bayesian optimization loop to optimize the Rosenbrock function.

API DOCUMENTATION

4.1 OMLT Block

The omlt.block module contains the implementation of the OmltBlock class. This class is used in combination with a formulation object to construct the necessary constraints and variables to represent ML models.

Example

```
import tensorflow.keras as keras
from omlt import OmltBlock
from omlt.neuralnet import FullSpaceNNFormulation
from omlt.io import load_keras_sequential

nn = keras.models.load_model(keras_fname)
net = load_keras_sequential(nn)

m = pyo.ConcreteModel()
m.neural_net_block = OmltBlock()
m.neural_net_block.build_formulation(FullSpaceNNFormulation(net))

m.obj = pyo.Objective(expr=(m.neural_net_block.outputs[2]-4.0)**2)
status = pyo.SolverFactory('ipopt').solve(m, tee=True)
pyo.assert_optimal_termination(status)

class omlt.block.OmltBlock(*args, **kwds)
    Bases: pyomo.core.base.block.CustomBlock
```

Note: *OmltBlock* is the name used to declare the custom Pyomo block which is exposed to the user. The block functionality is given by *OmltBlockData* which inherits from Pyomo *_BlockData*.

```
class omlt.block.OmltBlockData(component)
    Bases: pyomo.core.base.block._BlockData

    _setup_inputs_outputs(*, input_indexes, output_indexes)
        This function should be called by the derived class to create the inputs and outputs on the block
```

Parameters

- **input_indexes** – list list of indexes (can be tuples) defining the set to be used for the input variables

- **output_indexes** – list list of indexes (can be tuples) defining the set to be used for the input variables

build_formulation(formulation)

Call this method to construct the constraints (and possibly intermediate variables) necessary for the particular neural network formulation. The formulation object can be accessed later through the “formulation” attribute.

Parameters **formulation** (*instance of _PyomoFormulation*) – see, for example,
FullSpaceNNFormulation

4.2 Scaling

The omlt.scaling module describes the interface for providing different scaling expressions to the Pyomo model for the inputs and outputs of an ML model. An implementation of a common scaling approach is included with *OffsetScaling*.

class omlt.scaling.ScalingInterface

Bases: `abc.ABC`

abstract get_scaled_input_expressions(input_vars)

This method returns a list of expressions for the scaled inputs from the unscaled inputs

abstract get_unscaled_output_expressions(scaled_output_vars)

This method returns a list of expressions for the unscaled outputs from the scaled outputs

class omlt.scaling.OffsetScaling(offset_inputs, factor_inputs, offset_outputs, factor_outputs)

Bases: `omlt.scaling.ScalingInterface`

This scaling object represents the following scaling equations for inputs (x) and outputs (y)

$$x_i^{scaled} = \frac{(x_i - x_i^{offset})}{x_i^{factor}}$$

$$y_i^{scaled} = \frac{(y_i - y_i^{offset})}{y_i^{factor}}$$

Parameters

- **offset_inputs** (*array-like*) – Array of the values of the offsets for each input to the network
- **factor_inputs** (*array-like*) – Array of the scaling factors (division) for each input to the network
- **offset_outputs** (*array-like*) – Array of the values of the offsets for each output from the network
- **factor_outputs** (*array-like*) – Array of the scaling factors (division) for each output from the network

get_scaled_input_expressions(input_vars)

Get the scaled input expressions of the input variables.

get_scaled_output_expressions(output_vars)

Get the scaled output expressions of the output variables.

get_unscaled_input_expressions(scaled_input_vars)

Get the unscaled input expressions of the scaled input variables.

get_unscaled_output_expressions(*scaled_output_vars*)
Get the unscaled output expressions of the scaled output variables.

4.3 File IO

4.3.1 Input Bounds

`omlt.io.input_bounds.load_input_bounds(input_bounds_filename)`
Read the input bounds from the given file.

`omlt.io.input_bounds.write_input_bounds(input_bounds_filename, input_bounds)`
Write the specified input bounds to the given file.

4.3.2 Keras Reader

`omlt.io.keras_reader.load_keras_sequential(nn, scaling_object=None, scaled_input_bounds=None)`
Load a keras neural network model (built with Sequential) into an OMLT network definition object. This network definition object can be used in different formulations.

Parameters

- **nn** (`keras.model`) – A keras model that was built with Sequential
- **scaling_object** (*instance of ScalingInterface or None*) – Provide an instance of a scaling object to use to scale inputs → scaled_inputs and scaled_outputs → outputs. If None, no scaling is performed. See scaling.py.
- **scaled_input_bounds** (`dict` or `None`) – A dict that contains the bounds on the scaled variables (the direct inputs to the neural network). If None, then no bounds are specified.

Return type `NetworkDefinition`

4.3.3 ONNX

`omlt.io.onnx.load_onnx_neural_network(onnx, scaling_object=None, input_bounds=None)`
Load a NetworkDefinition from an onnx object.

Parameters

- **onnx** – onnx model
- **scaling_object** (*instance of object supporting ScalingInterface*) –
- **input_bounds** (*list of tuples*) –

Return type `NetworkDefinition`

`omlt.io.onnx.load_onnx_neural_network_with_bounds(filename)`
Load a NetworkDefinition with input bounds from an onnx object.

Parameters `filename` (`str`) – the path where the ONNX model and input bounds file are written

Return type `NetworkDefinition`

`omlt.io.onnx.write_onnx_model_with_bounds(filename, onnx_model=None, input_bounds=None)`
Write the ONNX model to the given file.

If `input_bounds` is not None, write it alongside the ONNX model.

Parameters

- **filename** (`str`) – the path where the ONNX model is written
- **onnx_model** (`onnx model or None`) – the onnx model
- **input_bounds** (`None or dict-like or list`) – bounds on the input variables

```
class omlt.io.onnx_parser.NetworkParser  
Bases: object
```

References

- <https://github.com/onnx/onnx/blob/master/docs/Operators.md>

```
parse_network(graph, scaling_object, input_bounds)
```

4.4 Gradient Boosted Trees

```
class omlt.gbt.gbt_formulation.GBTBigMFormulation(gbt_model)  
Bases: omlt.formulation._PyomoFormulation
```

property input_indexes

Return the indices corresponding to the inputs of the ML model. This is a list of entries (which may be tuples for higher dimensional inputs).

property output_indexes

Return the indices corresponding to the outputs of the ML model. This is a list of entries (which may be tuples for higher dimensional outputs).

```
omlt.gbt.gbt_formulation.add_formulation_to_block(block, model_definition, input_vars, output_vars)
```

References

- Misic, V. “Optimization of tree ensembles.” Operations Research 68.5 (2020): 1605-1624.
- Mistry, M., et al. “Mixed-integer convex nonlinear optimization with gradient-boosted trees embedded.” INFORMS Journal on Computing (2020).

```
class omlt.gbt.model.GradientBoostedTreeModel(onnx_model, scaling_object=None,  
                                              scaled_input_bounds=None)
```

Bases: object

property n_hidden

property n_inputs

property n_outputs

property onnx_model

property scaled_input_bounds

Return a list of tuples containing lower and upper bounds of neural network inputs

property scaling_object

Return an instance of the scaling object that supports the ScalingInterface

4.5 Neural Networks

4.5.1 OMLT Layers

Neural network layer classes.

```
class omlt.neuralnet.layer.ConvLayer(input_size, output_size, strides, kernel, *, activation=None,  
                                     input_index_mapper=None)
```

Bases: *omlt.neuralnet.layer.Layer*

Two-dimensional convolutional layer.

Parameters

- ***input_size*** (*tuple*) – the size of the input.
- ***output_size*** (*tuple*) – the size of the output.
- ***strides*** (*matrix-like*) – stride of the cross-correlation kernel.
- ***kernel*** (*matrix-like*) – the cross-correlation kernel.
- ***activation*** (*str* or *None*) – activation function name
- ***input_index_mapper*** (*IndexMapper* or *None*) – map indexes from this layer index to the input layer index size

property *kernel*

property *kernel_shape*

kernel_with_input_indexes(*out_d*, *out_r*, *out_c*)

Returns an iterator over the kernel value and input index for the output at index (*out_d*, *out_r*, *out_c*).

Parameters

- ***out_d*** (*int*) – the output depth.
- ***out_d*** – the output row.
- ***out_c*** (*int*) – the output column.

property *strides*

```
class omlt.neuralnet.layer.DenseLayer(input_size, output_size, weights, biases, *, activation=None,  
                                      input_index_mapper=None)
```

Bases: *omlt.neuralnet.layer.Layer*

Dense layer implementing $output = activation(dot(input, weights) + biases)$.

Parameters

- ***input_size*** (*tuple*) – the size of the input.
- ***output_size*** (*tuple*) – the size of the output.
- ***weight*** (*matrix-like*) – the weight matrix.
- ***biases*** (*array-like*) – the biases.
- ***activation*** (*str* or *None*) – activation function name
- ***input_index_mapper*** (*IndexMapper* or *None*) – map indexes from this layer index to the input layer index size

property *biases*

```
property weights

class omlt.neuralnet.layer.IndexMapper(input_size, output_size)
Bases: object

Map indexes from one layer to the other.

Parameters
• input_size (tuple) – the input size
• output_size (tuple) – the mapped input layer's output size

property input_size

property output_size

class omlt.neuralnet.layer.InputLayer(size)
Bases: omlt.neuralnet.layer.Layer

The first layer in any network.

Parameters size (tuple) – the size of the input.

class omlt.neuralnet.layer.Layer(input_size, output_size, *, activation=None, input_index_mapper=None)
Bases: object

Base layer class.

Parameters
• input_size (tuple) – size of the layer input
• output_size (tuple) – size of the layer output
• activation (str or None) – activation function name
• input_index_mapper (IndexMapper or None) – map indexes from this layer index to
the input layer index size

property activation
Return the activation function

eval(x)
Evaluate the layer at x.

Parameters x (array-like) – the input tensor. Must have size self.input_size.

property input_index_mapper
Return the index mapper

property input_indexes
Return a list of the input indexes

property input_indexes_with_input_layer_indexes
Return an iterator generating a tuple of local and input indexes.

Local indexes are indexes over the elements of the current layer. Input indexes are indexes over the elements
of the previous layer.

property input_size
Return the size of the input tensor

property output_indexes
Return a list of the output indexes
```

property output_size
Return the size of the output tensor

4.5.2 Neural Network Definition

```
class omlt.neuralnet.network_definition.NetworkDefinition(scaling_object=None,
                                                       scaled_input_bounds=None)
```

Bases: `object`

add_edge(*from_layer*, *to_layer*)
Add an edge between two layers.

Parameters

- **from_layer** (`Layer`) – the layer with the outbound connection
- **to_layer** (`Layer`) – the layer with the inbound connection

add_layer(*layer*)
Add a layer to the network.

Parameters `layer` (`Layer`) – the layer to add to the network

property input_layers
Return an iterator over the input layers

property input_nodes
An alias for `input_layers`

layer(*layer_id*)
Return the layer with the given id

property layers
Return an iterator over all the layers

property output_layers
Return an iterator over the output layer

property output_nodes
An alias for `output_layers`

predecessors(*layer*)
Return an iterator over the layers with outbound connections into the layer

property scaled_input_bounds
Return a dict of tuples containing lower and upper bounds of neural network inputs

property scaling_object
Return an instance of the scaling object that supports the `ScalingInterface`

successors(*layer*)
Return an iterator over the layers with an inbound connection from the layer

4.5.3 Neural Network Formulations

Base Formulation

```
class omlt.formulation._PyomoFormulation
Bases: omlt.formulation._PyomoFormulationInterface
```

This is a base class for different Pyomo formulations. To create a new formulation, inherit from this class and implement the abstract methods and properties.

property block

The underlying block containing the constraints / variables for this formulation.

abstract property input_indexes

Return the indices corresponding to the inputs of the ML model. This is a list of entries (which may be tuples for higher dimensional inputs).

abstract property output_indexes

Return the indices corresponding to the outputs of the ML model. This is a list of entries (which may be tuples for higher dimensional outputs).

Provided Formulations

```
class omlt.neuralnet.nn_formulation.FullSpaceNNFormulation(network_structure,
                                                               layer_constraints=None,
                                                               activation_constraints=None)
```

Bases: *omlt.formulation._PyomoFormulation*

This class is the entry-point to build neural network formulations.

This class iterates over all nodes in the neural network and for each one them, generates the constraints to represent the layer and its activation function.

Parameters

- **network_structure** (`NetworkDefinition`) – the neural network definition
- **layer_constraints** (`dict-like or None`) – overrides the constraints generated for the specified layer types
- **activation_constraints** (`dict-like or None`) – overrides the constraints generated for the specified activation functions

property block

The underlying block containing the constraints / variables for this formulation.

property input_indexes

The indexes of the formulation inputs.

property output_indexes

The indexes of the formulation output.

```
class omlt.neuralnet.nn_formulation.ReducedSpaceNNFormulation(network_structure,
                                                               activation_functions=None)
```

Bases: *omlt.formulation._PyomoFormulation*

This class is used to build reduced-space formulations of neural networks.

Parameters

- **network_structure** (`NetworkDefinition`) – the neural network definition

- **activation_functions** (*dict-like or None*) – overrides the actual functions used for particular activations

property block

The underlying block containing the constraints / variables for this formulation.

property input_indexes

The indexes of the formulation inputs.

property output_indexes

The indexes of the formulation output.

```
class omlt.neuralnet.nn_formulation.FullSpaceSmoothNNFormulation(network_structure)
```

Bases: *omlt.neuralnet.nn_formulation.FullSpaceNNFormulation*

property block

The underlying block containing the constraints / variables for this formulation.

property input_indexes

The indexes of the formulation inputs.

property output_indexes

The indexes of the formulation output.

```
class omlt.neuralnet.nn_formulation.ReducedSpaceSmoothNNFormulation(network_structure)
```

Bases: *omlt.neuralnet.nn_formulation.ReducedSpaceNNFormulation*

This class is used to build reduced-space formulations of neural networks with smooth activation functions.

Parameters **network_structure** (*NetworkDefinition*) – the neural network definition

property block

The underlying block containing the constraints / variables for this formulation.

property input_indexes

The indexes of the formulation inputs.

property output_indexes

The indexes of the formulation output.

```
class omlt.neuralnet.nn_formulation.ReluBigMFormulation(network_structure)
```

Bases: *omlt.neuralnet.nn_formulation.FullSpaceNNFormulation*

property block

The underlying block containing the constraints / variables for this formulation.

property input_indexes

The indexes of the formulation inputs.

property output_indexes

The indexes of the formulation output.

```
class omlt.neuralnet.nn_formulation.ReluComplementarityFormulation(network_structure)
```

Bases: *omlt.neuralnet.nn_formulation.FullSpaceNNFormulation*

property block

The underlying block containing the constraints / variables for this formulation.

property input_indexes

The indexes of the formulation inputs.

property output_indexes

The indexes of the formulation output.

```
class omlt.neuralnet.nn_formulation.ReluPartitionFormulation(network_structure,
                                                               split_func=None)
Bases: omlt.formulation._PyomoFormulation
```

This class is used to build partition-based formulations of neural networks.

Parameters

- **network_structure** (`NetworkDefinition`) – the neural network definition
- **split_func** (`callable`) – the function used to compute the splits

property block

The underlying block containing the constraints / variables for this formulation.

property input_indexes

The indexes of the formulation inputs.

property output_indexes

The indexes of the formulation output.

4.5.4 Layer and Activation Functions

We use the following notation to describe layer and activation functions:

$$\begin{aligned}N &:= \text{Set of nodes (i.e. neurons in the neural network)} \\M_i &:= \text{Number of inputs to node } i \\z_i &:= \text{pre-activation value on node } i \\z_i &:= \text{post-activation value on node } i \\w_{ij} &:= \text{weight from input } j \text{ to node } i \\b_i &:= \text{bias value for node } i\end{aligned}$$

Layer Functions

```
omlt.neuralnet.layers.full_space.full_space_conv_layer(net_block, net, layer_block, layer)
```

```
omlt.neuralnet.layers.full_space.full_space_dense_layer(net_block, net, layer_block, layer)
```

Add full-space formulation of the dense layer to the block

$$\hat{z}_i = \sum_{j=1}^{M_i} w_{ij} z_j + b_i \quad \forall i \in N$$

```
omlt.neuralnet.layers.reduced_space.reduced_space_dense_layer(net_block, net, layer_block, layer,
                                                               activation)
```

Add reduced-space formulation of the dense layer to the block

$$\hat{z}_i = \sum_{j=1}^{M_i} w_{ij} z_j + b_i \quad \forall i \in N$$

```
omlt.neuralnet.layers.partition_based.default_partition_split_func(w, n)
```

```
omlt.neuralnet.layers.partition_based.partition_based_dense_relu_layer(net_block, net,
                                                                     layer_block, layer,
                                                                     split_func)
```

Activation Functions

```
omlt.neuralnet.activations.linear.linear_activation_constraint(net_block, net, layer_block, layer,
                                                               add_constraint=True)
```

Linear activation constraint generator

Generates the constraints for the linear activation function.

$$z_i = \hat{z}_i \quad \forall i \in N$$

```
omlt.neuralnet.activations.linear.linear_activation_function(zhat)
```

```
class omlt.neuralnet.activations.relu.ComplementarityReLUActivation(transform=None)
Bases: object
```

Complementarity-based ReLU activation formulation.

```
omlt.neuralnet.activations.relu.bigm_relu_activation_constraint(net_block, net, layer_block,
                                                               layer)
```

Big-M ReLU activation formulation.

```
omlt.neuralnet.activations.smooth.sigmoid_activation_constraint(net_block, net, layer_block,
                                                               layer)
```

Sigmoid activation constraint generator

Generates the constraints for the sigmoid activation function.

$$z_i = \frac{1}{1 + \exp(-\hat{z}_i)} \quad \forall i \in N$$

```
omlt.neuralnet.activations.smooth.sigmoid_activation_function(x)
```

```
omlt.neuralnet.activations.smooth.smooth_monotonic_activation_constraint(net_block, net,
                                                                       layer_block, layer,
                                                                       fcn)
```

Activation constraint generator for a smooth monotonic function

Generates the constraints for the activation function fcn if it is smooth and monotonic

$$z_i = fcn(\hat{z}_i) \quad \forall i \in N$$

```
omlt.neuralnet.activations.smooth.softplus_activation_constraint(net_block, net, layer_block,
                                                               layer)
```

Softplus activation constraint generator

Generates the constraints for the softplus activation function.

$$z_i = \log(\exp(\hat{z}_i) + 1) \quad \forall i \in N$$

```
omlt.neuralnet.activations.smooth.softplus_activation_function(x)
```

```
omlt.neuralnet.activations.smooth.tanh_activation_constraint(net_block, net, layer_block, layer)
tanh activation constraint generator
```

Generates the constraints for the tanh activation function.

$$z_i = \tanh(\hat{z}_i) \quad \forall i \in N$$

```
omlt.neuralnet.activations.smooth.tanh_activation_function(x)
```


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